

## Uitwerkingen hoofdstuk 4

### 4.3

1. a.  $(3x - 1)^2 = (3x)^2 - 2 \cdot 3x \cdot 1 + 1^2 = 9x^2 - 6x + 1$   
b.  $(a + 2b)^2 = a^2 + 2 \cdot a \cdot 2b + (2b)^2 = a^2 + 4ab + 4b^2$   
c.  $(2x + 1)(2x - 1) = (2x)^2 - 1^2 = 4x^2 - 1$   
d.  $(4x - 3)(3x + 4) = 4x(3x + 4) - 3(3x + 4) = 12x^2 + 16x - 9x - 12 = 12x^2 + 7x - 12$   
e.  $2x(3x - 1) - 2(x^2 + 3) = 6x^2 - 2x - 2x^2 - 6 = 4x^2 - 2x - 6$   
f.  $(1 + \sqrt{2})(1 - \sqrt{2}) = 1^2 - (\sqrt{2})^2 = 1 - 2 = -1$
  
2. a.  $(3x - p)^2 = (3x)^2 - 2 \cdot 3x \cdot p + p^2 = 9x^2 - 6xp + p^2$   
b.  $(a + 2\sqrt{b})^2 = a^2 + 2 \cdot a \cdot 2\sqrt{b} + (2\sqrt{b})^2 = a^2 + 4a\sqrt{b} + 4b$   
c.  $(2x + y)(2x - y) = (2x)^2 - y^2 = 4x^2 - y^2$   
d.  $(\omega - 3)(\omega + 4) = \omega(\omega + 4) - 3(\omega + 4) = \omega^2 + 4\omega - 3\omega - 12 = \omega^2 + \omega - 12$   
e.  $x(3x - 1) + 2(x^2 - 3) = 3x^2 - x + 2x^2 - 6 = 5x^2 - x - 6$   
f.  $(t + \sqrt{3})(t - \sqrt{3}) = t^2 - (\sqrt{3})^2 = t^2 - 3$
  
3. a.  $\frac{1}{1 - \sqrt{2}} = \frac{1 + \sqrt{2}}{(1 - \sqrt{2})(1 + \sqrt{2})} = \frac{1 + \sqrt{2}}{-1} = -1 - \sqrt{2}$   
b.  $\frac{1}{\sqrt{3} - 1} = \frac{\sqrt{3} + 1}{(\sqrt{3} - 1)(\sqrt{3} + 1)} = \frac{\sqrt{3} + 1}{2} = \frac{1}{2} + \frac{1}{2}\sqrt{3}$   
c.  $\frac{1}{\sqrt{2} + \sqrt{3}} = \frac{\sqrt{2} - \sqrt{3}}{(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})} = \frac{\sqrt{2} - \sqrt{3}}{-1} = \sqrt{3} - \sqrt{2}$   
d.  $\frac{1}{2\sqrt{3} - 1} = \frac{2\sqrt{3} + 1}{(2\sqrt{3} - 1)(2\sqrt{3} + 1)} = \frac{2\sqrt{3} + 1}{12 - 1} = \frac{1}{11} + \frac{2}{11}\sqrt{3}$

4. a.  $2a(3a + 4a) = 2a \cdot 7a = 14a^2$   
 b.  $(3t - 4t)5t = -t \cdot 5t = -5t^2$   
 c.  $-2(a - 3b + c) = -2a + 6b - 2c$   
 d.  $a - 2c - (a + 6b - c\sqrt{3}) = a - 2c - a - 6b + c\sqrt{3} = -6b - 2c + \sqrt{3} \cdot c = -6b - (2 - \sqrt{3})c$   
 e.  $(3a + 4b)(1 - a) = 3a(1 - a) + 4b(1 - a) = 3a - 3a^2 + 4b - 4ab = -3a^2 + 3a - 4ab + 4b$   
 f.  $(3 - 2p^2)(p^2 - 4) = 3(p^2 - 4) - 2p^2(p^2 - 4) = 3p^2 - 12 - 2p^4 + 8p^2 = -2p^4 + 11p^2 - 12$
5. a.  $\frac{2A}{x+3} - \frac{A}{x} = \frac{2Ax}{x(x+3)} - \frac{A(x+3)}{x(x+3)} = \frac{2Ax - Ax - 3A}{x(x+3)} = \frac{Ax - 3A}{x(x+3)} = \frac{A(x-3)}{x(x+3)}$   
 b.  $\frac{4t}{t+5} - \frac{1}{t-5} = \frac{4t(t-5)}{(t+5)(t-5)} - \frac{t+5}{(t+5)(t-5)} = \frac{4t^2 - 20t - t - 5}{(t+5)(t-5)} = \frac{4t^2 - 21t - 5}{(t+5)(t-5)}$   
 c.  $\omega + \frac{\omega}{t+1} = \frac{\omega(t+1)}{t+1} + \frac{\omega}{t+1} = \frac{\omega t + 2\omega}{t+1} = \frac{\omega(t+2)}{t+1}$   
 d.  $\frac{1}{x+1} + \frac{3x}{(x+1)^2} = \frac{x+1}{(x+1)^2} + \frac{3x}{(x+1)^2} = \frac{4x+1}{(x+1)^2}$   
 e.  $\frac{3}{2x+1} - \frac{1}{(2x+1)^3} = \frac{3(2x+1)^2}{(2x+1)^3} - \frac{1}{(2x+1)^3} = \frac{3(4x^2 + 4x + 1) - 1}{(2x+1)^3} = \frac{12x^2 + 12x + 2}{(2x+1)^3}$   
 f.  $\sqrt{t+1} - \frac{A}{\sqrt{t+1}} = \frac{(\sqrt{t+1})^2}{\sqrt{t+1}} - \frac{A}{\sqrt{t+1}} = \frac{t+1-A}{\sqrt{t+1}}$
6. a.  $x^3 - 9x^2 = x^2(x - 9)$   
 b.  $t^3 - 9t = t(t^2 - 9) = t(t-3)(t+3)$   
 c.  $r^3 - 5r = r(r^2 - 5) = r(r - \sqrt{5})(r + \sqrt{5})$   
 d.  $a^2 - \frac{1}{4} = (a + \frac{1}{2})(a - \frac{1}{2})$   
 e.  $x^2 + 7x + 12 = (x + 3)(x + 4)$   
 f.  $h^2 - 7h + 12 = (h - 3)(h - 4)$
7. a.  $x^2 - x - 12 = (x - 4)(x + 3)$   
 b.  $a^2 + a - 12 = (a + 4)(a - 3)$   
 c.  $t^2 - 9 = (t - 3)(t + 3)$   
 d.  $2t^2 - 8 = 2(t^2 - 4) = 2(t - 2)(t + 2)$   
 e.  $r^4 - 1 = (r^2 + 1)(r^2 - 1) = (r^2 + 1)(r + 1)(r - 1)$   
 f.  $3x^2 - 12x - 63 = 3(x^2 - 4x - 21) = 3(x - 7)(x + 3)$

8. a.  $x^2 - 19x + 34 = (x - 2)(x - 17)$
- b.  $2x^2 - 24 = 2(x^2 - 12) = 2(x - \sqrt{12})(x + \sqrt{12}) = 2(x - 2\sqrt{3})(x + 2\sqrt{3})$
- c.  $4x^2 - 32 = 4(x^2 - 8) = 4(x - 2\sqrt{2})(x + 2\sqrt{2})$
- d.  $9x^2 + 27x - 90 = 9(x^2 + 3x - 10) = 9(x + 5)(x - 2)$
- e.  $3x^2 + 12x - 63 = 3(x^2 + 4x - 21) = 3(x + 7)(x - 3)$
- f.  $x^3 + 2x^2 - 24x = x(x^2 + 2x - 24) = x(x + 6)(x - 4)$
- g.  $6x^2 - 30x + 24 = 6(x^2 - 5x + 4) = 6(x - 4)(x - 1)$
- h.  $x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4) = (x + 1)(x - 1)(x + 2)(x - 2)$
- i.  $2x^4 - 32 = 2(x^4 - 16) = 2(x^2 + 4)(x^2 - 4) = 2(x^2 + 4)(x + 2)(x - 2)$
- j.  $x(x + 4) + x + 4 = x(x + 4) + 1 \cdot (x + 4) = (x + 1)(x + 4)$
- k.  $x - 1 + 6(x^2 - 1) = 1 \cdot (x - 1) + 6(x + 1)(x - 1) = (1 + 6(x + 1))(x - 1) = (6x + 7)(x - 1)$
- l.  $x - 1 - 2(x^2 - 1) = (x - 1) - 2(x + 1)(x - 1) = (1 - 2(x + 1))(x - 1) = (-2x - 1)(x - 1)$   
 $= -(2x + 1)(x - 1)$

$$\begin{aligned}
 9. (a + b)^3 &= (a + b)(a + b)^2 \\
 &= (a + b)(a^2 + 2ab + b^2) \\
 &= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3 \\
 &= a^3 + 3a^2b + 3ab^2 + b^3
 \end{aligned}$$

en evenzo (of vul voor  $b$  in het bovenstaande resultaat  $-b$  in):

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

10. Uitwerken van de rechterkant geeft:

$$(a - b)(a^2 + ab + b^2) = a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3 = a^3 - b^3$$