

### Uitwerkingen Hoofdstuk 8

1. a.  $\frac{2 \cdot 4 \cdot 6}{4} = 2 \cdot 6 = 12$   
b.  $\frac{2 + 3 + 6}{4} = \frac{11}{4} = 2\frac{3}{4}$   
c.  $\frac{21 \cdot 6}{7 \cdot 3} = \frac{7 \cdot 3 \cdot 6}{7 \cdot 3} = 6$   
d.  $\frac{21 \cdot 6}{7 + 3} = \frac{21 \cdot 6}{10} = \frac{21 \cdot 3 \cdot 2}{5 \cdot 2} = \frac{63}{5} = 12\frac{3}{5}$   
e.  $-\frac{8 - 10}{-2} = -\frac{-2}{-2} = -1$   
f.  $\frac{23 \cdot -2}{-1} = \frac{-46}{-1} = 46$
  
2. a.  $3\frac{1}{4} + 4\frac{1}{5} = 3\frac{5}{20} + 4\frac{4}{20} = 7\frac{9}{20}$   
b.  $3\frac{1}{4} - 4\frac{1}{5} = 3\frac{5}{20} - 4\frac{4}{20} = 3\frac{5}{20} - 3\frac{24}{20} = -\frac{19}{20}$   
c.  $3\frac{1}{4} \cdot 4\frac{1}{5} = \frac{13}{4} \cdot \frac{21}{5} = \frac{273}{20} = 13\frac{13}{20}$   
d.  $3\frac{1}{4} : 4\frac{1}{5} = \frac{13}{4} : \frac{21}{5} = \frac{13}{4} \cdot \frac{5}{21} = \frac{65}{84}$   
e.  $\frac{3}{5} : \frac{1}{5} = \frac{3}{5} \cdot \frac{5}{1} = 3$   
f.  $\frac{3}{5} : 5 = \frac{3}{5} \cdot \frac{1}{5} = \frac{3}{25}$
  
3. a.  $\frac{x}{x-1} - \frac{1}{x+1} = \frac{x(x+1)}{(x-1)(x+1)} - \frac{x-1}{(x-1)(x+1)} = \frac{x^2 + x - x + 1}{(x-1)(x+1)} = \frac{x^2 + 1}{x^2 - 1}$   
b.  $\frac{1}{3} - \frac{x}{2x-1} = \frac{2x-1}{3(2x-1)} - \frac{3x}{3(2x-1)} = \frac{2x-1-3x}{3(2x-1)} = \frac{-x-1}{3(2x-1)} = -\frac{x+1}{3(2x-1)}$   
c.  $\frac{1}{\sqrt{p}} - \frac{1}{\sqrt{q}} = \frac{\sqrt{q}}{\sqrt{pq}} - \frac{\sqrt{p}}{\sqrt{pq}} = \frac{\sqrt{q} - \sqrt{p}}{\sqrt{pq}}$   
d.  $\frac{1}{a+b} + \frac{b}{a^2-b^2} = \frac{a-b}{a^2-b^2} + \frac{b}{a^2-b^2} = \frac{a}{a^2-b^2}$   
e.  $t + \frac{1}{t} + \frac{1}{t^2} = \frac{t^3}{t^2} + \frac{t}{t^2} + \frac{1}{t^2} = \frac{t^3 + t + 1}{t^2}$   
f.  $\frac{1}{xy} - \frac{y}{x} + \frac{x}{y} = \frac{1}{xy} - \frac{y^2}{xy} + \frac{x^2}{xy} = \frac{x^2 - y^2 + 1}{xy}$
  
4. a.  $\frac{1}{p} + \frac{1}{q} = \frac{q}{pq} + \frac{p}{pq} = \frac{p+q}{pq}$   
b.  $-a\left(\frac{1}{p} - \frac{1}{q}\right) = -a\left(\frac{q}{pq} - \frac{p}{pq}\right) = -a\left(\frac{q-p}{pq}\right) = \frac{a(p-q)}{pq}$   
c.  $\frac{1}{p} \cdot \frac{1}{q} = \frac{1}{pq}$   
d.  $\frac{1}{p} : \frac{1}{q} = \frac{1}{p} \cdot \frac{q}{1} = \frac{q}{p}$

$$e. \frac{1}{\frac{1}{p} + 1} = \frac{p \cdot 1}{p \cdot \left(\frac{1}{p} + 1\right)} = \frac{p}{1 + p} = \frac{p}{p + 1}$$

$$f. \frac{1}{\frac{1}{p} : \frac{1}{q}} = \frac{1}{\frac{1}{p} \cdot \frac{q}{1}} = \frac{1}{\left(\frac{q}{p}\right)} = 1 \cdot \frac{p}{q} = \frac{p}{q}$$

$$5. a. \frac{1}{x+1} : \frac{1}{x-1} = \frac{1}{x+1} \cdot \frac{x-1}{1} = \frac{x-1}{x+1}$$

$$b. \frac{1}{x+1} \cdot \frac{x}{x-1} = \frac{x}{(x+1)(x-1)} = \frac{x}{x^2-1}$$

$$c. \frac{1}{\sqrt{3}} : \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{6}}{1} = \frac{\sqrt{6}}{\sqrt{3}} = \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{3}} = \sqrt{2}$$

$$d. \frac{abx}{a^2xy^2} \cdot y = \frac{abxy}{a^2xy^2} = \frac{b}{ay}$$

$$e. v(1-2v) + v(2v-1) = v - 2v^2 + 2v^2 - v = 0$$

$$f. \frac{2}{3} - \frac{a}{a+b} = \frac{2(a+b)}{3(a+b)} - \frac{3a}{3(a+b)} = \frac{2a+2b-3a}{3(a+b)} = \frac{2b-a}{3(a+b)}$$

$$6. a. 2\sqrt{8} - 8\sqrt{2} = 2 \cdot 2\sqrt{2} - 8\sqrt{2} = 4\sqrt{2} - 8\sqrt{2} = -4\sqrt{2}$$

$$b. 14\sqrt{8} \cdot 2\sqrt{2} = 14 \cdot 2\sqrt{2} \cdot 2\sqrt{2} = 14 \cdot 4 \cdot 2 = 112$$

$$c. 16^{\frac{2}{3}} = (2^4)^{\frac{2}{3}} = 2^{4 \cdot \frac{2}{3}} = 2^{\frac{8}{3}} = 2^{2\frac{2}{3}} = 2^2 \sqrt[3]{2^2} = 4\sqrt[3]{4}$$

$$d. \sqrt[5]{16^2} = 16^{\frac{2}{5}} = (2^4)^{\frac{2}{5}} = 2^{4 \cdot \frac{2}{5}} = 2^{\frac{8}{5}} = 2^{1\frac{3}{5}} = 2 \cdot 2^{\frac{3}{5}} = 2 \sqrt[5]{2^3} = 2\sqrt[5]{8}$$

$$e. \left(\frac{1}{3}\right)^{-\frac{2}{3}} = (3^{-1})^{-\frac{2}{3}} = 3^{\frac{2}{3}} = \sqrt[3]{3^2} = \sqrt[3]{9}$$

$$f. (-2\sqrt[3]{4})^4 = (-2)^4 \cdot (\sqrt[3]{4})^4 = 2^4 \cdot \left(2^{\frac{2}{3}}\right)^4 = 2^4 \cdot 2^{\frac{8}{3}} = 2^4 \cdot 2^2 \sqrt[3]{2^2} = 64\sqrt[3]{4}$$

$$7. a. \left(\frac{6}{\sqrt[3]{p}}\right)^4 = \left(\frac{6}{p^{\frac{1}{3}}}\right)^4 = \frac{6^4}{p^{\frac{4}{3}}} = \frac{6^4}{p^{1\frac{1}{3}}} = \frac{6^4}{p\sqrt[3]{p}} = \frac{1296}{p\sqrt[3]{p}}$$

$$b. \left(\frac{1}{\sqrt[3]{p}}\right)^6 = \left(p^{-\frac{1}{3}}\right)^6 = p^{-\frac{6}{3}} = p^{-2} = \frac{1}{p^2} = \frac{1}{p\sqrt{p}}$$

$$c. 2\frac{p^2}{\sqrt[3]{p}} = 2\frac{p^2}{p^{\frac{1}{3}}} = 2p^{2-\frac{1}{3}} = 2p^{1\frac{2}{3}} = 2p\sqrt[3]{p^2}$$

$$d. \frac{3p^3 + 2p\sqrt{p}}{p\sqrt{3p^3}} = \frac{3p^2 + 2\sqrt{p}}{\sqrt{3p^3}} = \frac{3p^2 + 2\sqrt{p}}{p\sqrt{p} \cdot \sqrt{3}} = \frac{3p^2}{p\sqrt{p} \cdot \sqrt{3}} + \frac{2\sqrt{p}}{p\sqrt{p} \cdot \sqrt{3}} = \sqrt{3p} + \frac{2}{p\sqrt{3}}$$

$$8. \sqrt[p]{\sqrt[q]{x}} = \sqrt[p]{x^{\frac{1}{q}}} = \left(x^{\frac{1}{q}}\right)^{\frac{1}{p}} = x^{\frac{1}{p} \cdot \frac{1}{q}} = x^{\frac{1}{pq}}$$

$$\sqrt[3]{4} \cdot \sqrt{\sqrt[3]{4}} = 4^{\frac{1}{3}} \cdot 4^{\frac{1}{6}} = 4^{\frac{1}{3} + \frac{1}{6}} = 4^{\frac{1}{2}} = \sqrt{4} = 2$$

$$9. \text{ a. } (3x - y)^2 = (3x)^2 - 2 \cdot 3x \cdot y + y^2 = 9x^2 - 6xy + y^2$$

$$\text{ b. } (a + b)32 = 32a + 32b$$

$$\text{ c. } (2x + 1)(2x - 1)(4x^2 + 1) = (4x^2 - 1)(4x^2 + 1) = 16x^4 - 1$$

$$\text{ d. } (4x - \sqrt{3})(4x + \sqrt{3}) = 16x^2 - 3$$

$$\text{ e. } \sqrt{2x+1} - \frac{2x}{\sqrt{2x+1}} = \frac{2x+1}{\sqrt{2x+1}} - \frac{2x}{\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

$$\text{ f. } \frac{1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{\sqrt{5}+1}{5-1} = \frac{1+\sqrt{5}}{4} = \frac{1}{4} + \frac{1}{4}\sqrt{5}$$

$$10. \text{ a. } x^3 - x^2 = x^2(x - 1)$$

$$\text{ b. } t^3 - t = t(t^2 - 1) = t(t+1)(t-1)$$

$$\text{ c. } t^2 + 5t + 6 = (t+2)(t+3)$$

$$\text{ d. } 2t^2 - 8t + 6 = 2(t^2 - 4t + 3) = 2(t-1)(t-3)$$

$$\text{ e. } 1 - t^4 = (1 - t^2)(1 + t^2) = (1 - t)(1 + t)(1 + t^2)$$

$$\text{ f. } 3v^2 - 12v - 63 = 3(v^2 - 4v - 21) = 3(v-7)(v+3)$$

$$11. \text{ a. } \quad 2x + 3p = -x + 6 \quad \{-x \text{ naar links, } 3p \text{ naar rechts}\}$$

$$\Leftrightarrow 3x = 6 - 3p \quad \{\text{deel door } 3\}$$

$$\Leftrightarrow x = 2 - p$$

$$\text{ b. } \quad \frac{1}{2}xp = \frac{x-1}{4} \quad \{\text{maal } 4\}$$

$$\Leftrightarrow 2xp = x - 1 \quad \{x \text{ naar links}\}$$

$$\Leftrightarrow 2xp - x = -1 \quad \{x \text{ buiten haakjes halen}\}$$

$$\Leftrightarrow x(2p - 1) = -1 \quad \{\text{deel door } 2p - 1\}$$

$$\Leftrightarrow x = \frac{-1}{2p-1} = \frac{1}{1-2p}$$

$$\text{ c. } \quad 4x - a = 3x - b \quad \{3x \text{ naar links, } -a \text{ naar rechts}\}$$

$$\Leftrightarrow x = a - b$$

$$\begin{aligned}
\text{d.} \quad & a(x-1) = \frac{x+1}{a} && \{\text{maal } a, \text{ voorwaarde } a \neq 0\} \\
\Leftrightarrow & a^2(x-1) = x+1 && \{\text{haakjes uitwerken}\} \\
\Leftrightarrow & a^2x - a^2 = x+1 && \{x \text{ naar links, } -a^2 \text{ naar rechts}\} \\
\Leftrightarrow & a^2x - x = a^2 + 1 && \{x \text{ buiten haakjes halen}\} \\
\Leftrightarrow & x(a^2 - 1) = a^2 + 1 && \{\text{deel door } a^2 - 1\} \\
\Leftrightarrow & x = \frac{a^2 + 1}{a^2 - 1}, \text{ mits } a \neq 0
\end{aligned}$$

De oplossing is  $x = \frac{a^2 + 1}{a^2 - 1}$ , voor  $a \neq 0$ .

Uiteraard mag de noemer niet 0 zijn en is er ook geen oplossing voor  $a = 1$  of  $a = -1$ , maar omdat je dat aan het antwoord kunt zien, schrijf je die voorwaarden er niet bij.

$$\begin{aligned}
\text{e.} \quad & (x-a)(x-b) = x(x+1) && \{\text{haakjes uitwerken}\} \\
\Leftrightarrow & x^2 - (a+b)x + ab = x^2 + x && \{x^2 + x \text{ naar links, } ab \text{ naar rechts}\} \\
\Leftrightarrow & -x^2 - x + x^2 - (a+b)x = -ab \\
\Leftrightarrow & -x - (a+b)x = -ab \\
\Leftrightarrow & x + (a+b)x = ab \\
\Leftrightarrow & (1+a+b)x = ab \\
\Leftrightarrow & x = \frac{ab}{1+a+b}
\end{aligned}$$

$$\begin{aligned}
\text{f.} \quad & a(2x+1) = \frac{b}{2x-1} && \{\text{maal } 2x-1, \text{ voorwaarde } x \neq \frac{1}{2}\} \\
\Leftrightarrow & a(2x+1)(2x-1) = b && \{\text{haakjes uitwerken}\} \\
\Leftrightarrow & a(4x^2 - 1) = b && \{\text{deel door } a\} \\
\Leftrightarrow & 4x^2 - 1 = \frac{b}{a} && \{-1 \text{ naar rechts}\} \\
\Leftrightarrow & 4x^2 = \frac{a+b}{a} && \{\text{deel door } 4\} \\
\Leftrightarrow & x^2 = \frac{a+b}{4a} && \{\text{standaardvergelijking}\} \\
\Leftrightarrow & x = \sqrt{\frac{a+b}{4a}} \vee x = -\sqrt{\frac{a+b}{4a}} \\
\Leftrightarrow & x = \frac{1}{2}\sqrt{\frac{a+b}{a}} \vee x = -\frac{1}{2}\sqrt{\frac{a+b}{a}}, \text{ mits } x \neq \frac{1}{2}
\end{aligned}$$

Geldt bijvoorbeeld  $b = 0$ , dan vind je  $x = \frac{1}{2} \vee x = -\frac{1}{2}$  en de eerste hiervan voldoet niet.

12. a.  $3x - 1 \leq x + 4$   $\{x \text{ naar links, } -1 \text{ naar rechts}\}$   
 $\Leftrightarrow 2x \leq 5$   $\{\text{deel door } 2\}$   
 $\Leftrightarrow x \leq 2\frac{1}{2}$
- b.  $x - 4 \geq 3x + 4$   $\{3x \text{ naar links, } -4 \text{ naar rechts}\}$   
 $\Leftrightarrow -2x \geq 8$   $\{\text{deel door } -2, \text{ teken klapt om}\}$   
 $\Leftrightarrow x \leq -4$
- c.  $2(1 - 3x) > 4(x - 4)$   $\{\text{haakjes uitwerken}\}$   
 $\Leftrightarrow 2 - 6x > 4x - 16$   $\{4x \text{ naar links, } 2 \text{ naar rechts}\}$   
 $\Leftrightarrow -10x > -18$   $\{\text{deel door } -10, \text{ teken klapt om}\}$   
 $\Leftrightarrow x < 1\frac{4}{5}$
- d.  $1 + x\sqrt{3} < 7 - x\sqrt{3}$   $\{-x\sqrt{3} \text{ naar links, } 1 \text{ naar rechts}\}$   
 $\Leftrightarrow 2x\sqrt{3} < 6$   $\{\text{deel door } 2\sqrt{3}\}$   
 $\Leftrightarrow x < \frac{6}{2\sqrt{3}}$   $\{\text{vereenvoudig}\}$   
 $\Leftrightarrow x < \sqrt{3}$
- e.  $4(x - 1) < 4x - 1$   $\{\text{haakjes uitwerken}\}$   
 $\Leftrightarrow 4x - 4 < 4x - 1$   $\{4x \text{ naar links, } -4 \text{ naar rechts}\}$   
 $\Leftrightarrow 0 < 3$   $\{\text{dit geldt voor alle waarden van } x\}$   
 $\Leftrightarrow x \in \mathbb{R}$  (elke waarde van  $x$  voldoet)
- f.  $\frac{2}{3}x > \frac{1}{3}x + 4$   $\{\text{maal } 3\}$   
 $\Leftrightarrow 2x > x + 12$   $\{x \text{ naar links}\}$   
 $\Leftrightarrow x > 12$

13. a.  $2x^2 - 5x + 2 = 0$  {discriminant:  $(-5)^2 - 4 \cdot 2 \cdot 2 = 9$ }
- $$\Leftrightarrow x_{1,2} = \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4}$$
- $$\Leftrightarrow x = 2 \vee x = \frac{1}{2}$$
- b.  $2x^2 + 5x + 2 = 0$  {discriminant:  $5^2 - 4 \cdot 2 \cdot 2 = 9$ }
- $$\Leftrightarrow x_{1,2} = \frac{-5 \pm \sqrt{9}}{4} = \frac{-5 \pm 3}{4}$$
- $$\Leftrightarrow x = -2 \vee x = -\frac{1}{2}$$
- c.  $4x^2 - 5x\sqrt{2} + 2 = 0$  {discriminant:  $(-5\sqrt{2})^2 - 4 \cdot 4 \cdot 2 = 18$ }
- $$\Leftrightarrow x_{1,2} = \frac{5\sqrt{2} \pm \sqrt{18}}{8} = \frac{5\sqrt{2} \pm 3\sqrt{2}}{8}$$
- $$\Leftrightarrow x = \sqrt{2} \vee x = \frac{1}{4}\sqrt{2}$$
- d.  $4x^2 + 5x\sqrt{2} + 2 = 0$  {discriminant:  $(5\sqrt{2})^2 - 4 \cdot 4 \cdot 2 = 18$ }
- $$\Leftrightarrow x_{1,2} = \frac{-5\sqrt{2} \pm \sqrt{18}}{8} = \frac{-5\sqrt{2} \pm 3\sqrt{2}}{8}$$
- $$\Leftrightarrow x = -\sqrt{2} \vee x = -\frac{1}{4}\sqrt{2}$$
- e.  $2x^2 - 5x\sqrt{2} + 4 = 0$  {discriminant:  $(-5\sqrt{2})^2 - 4 \cdot 2 \cdot 4 = 18$ }
- $$\Leftrightarrow x_{1,2} = \frac{5\sqrt{2} \pm \sqrt{18}}{4} = \frac{5\sqrt{2} \pm 3\sqrt{2}}{4}$$
- $$\Leftrightarrow x = \frac{1}{2}\sqrt{2} \vee x = 2\sqrt{2}$$
- f.  $2x^2 + 5x\sqrt{2} + 4 = 0$  {discriminant:  $(5\sqrt{2})^2 - 4 \cdot 2 \cdot 4 = 18$ }
- $$\Leftrightarrow x_{1,2} = \frac{-5\sqrt{2} \pm \sqrt{18}}{4} = \frac{-5\sqrt{2} \pm 3\sqrt{2}}{4}$$
- $$\Leftrightarrow x = -\frac{1}{2}\sqrt{2} \vee x = -2\sqrt{2}$$

14. a.  $(2t - 1)^3 = -1$  {standaardvergelijking}
- $\Leftrightarrow 2t - 1 = \sqrt[3]{-1} = -1$
- $\Leftrightarrow 2t = 0$
- $\Leftrightarrow t = 0$
- b.  $(t - 1)^4 = 81$  {standaardvergelijking}
- $\Leftrightarrow t - 1 = \pm \sqrt[4]{81} = \pm 3$
- $\Leftrightarrow t = 1 \pm 3$
- $\Leftrightarrow t = 4 \vee t = -2$
- c.  $(t + \frac{1}{t})^2 = 4$  {standaardvergelijking}
- $\Leftrightarrow t + \frac{1}{t} = 2 \vee t + \frac{1}{t} = -2$  {links en rechts maal  $t$ , voorwaarde:  $t \neq 0$ }
- $\Leftrightarrow t^2 + 1 = 2t \vee t^2 + 1 = -2t$
- $\Leftrightarrow t^2 - 2t + 1 = 0 \vee t^2 + 2t + 1 = 0$
- $\Leftrightarrow (t - 1)^2 = 0 \vee (t + 1)^2 = 0$
- $\Leftrightarrow t = 1 \vee t = -1$
- d.  $(1 - t)^3 = A$  {standaardvergelijking}
- $\Leftrightarrow 1 - t = \sqrt[3]{A}$
- $\Leftrightarrow t = 1 - \sqrt[3]{A}$
- e.  $(t - a)(t - b) = a \cdot b$  {haakjes uitwerken}
- $\Leftrightarrow t^2 - (a + b)t + ab = ab$
- $\Leftrightarrow t^2 - (a + b)t = 0$
- $\Leftrightarrow t(t - (a + b)) = 0$
- $\Leftrightarrow t = 0 \vee t = a + b$
- f.  $t^6 + t^3 = 12$  {substitutie  $t^3 = x$ , equivalent met  $t = \sqrt[3]{x}$ }
- $\Leftrightarrow x^2 + x - 12 = 0$
- $\Leftrightarrow (x - 3)(x + 4) = 0$
- $\Leftrightarrow x = 3 \vee x = -4$  { $t = \sqrt[3]{x}$ }
- $\Leftrightarrow t = \sqrt[3]{3} \vee t = -\sqrt[3]{4}$

15. De discriminant van de vergelijking  $ax^2 + 6x + 1 = 0$  is  $36 - 4a$ . Hiervoor moet gelden:

$$\begin{aligned} 36 - 4a &< 0 && \{36 \text{ naar rechts}\} \\ \Leftrightarrow -4a &< -36 && \{\text{deel door } -4, \text{ teken klapt om}\} \\ \Leftrightarrow a &> 9 \end{aligned}$$

Dus voor  $a > 9$  heeft deze vergelijking geen oplossingen.

$$\begin{aligned} 16. \quad 2\omega &= 2 + \sqrt{2\omega + 10} && \{2 \text{ naar links}\} \\ \Leftrightarrow 2\omega - 2 &= \sqrt{2\omega + 10} && \{\text{kwadrateer, geen equivalentie!}\} \\ \Rightarrow 4\omega^2 - 8\omega + 4 &= 2\omega + 10 && \{\text{herleid tot } 0 \text{ en deel door } 2\} \\ \Leftrightarrow 2\omega^2 - 5\omega - 3 &= 0 && \{\text{discriminant is } 25 - 4 \cdot 2 \cdot -3 = 49\} \\ \Leftrightarrow \omega_{1,2} &= \frac{5 \pm 7}{4} \\ \Leftrightarrow \omega &= 3 \vee \omega = -\frac{1}{2} \end{aligned}$$

Controle:

$$\omega = 3 : 2 \cdot 3 = 2 + \sqrt{2 \cdot 3 + 10} \text{ levert: } 6 = 2 + 4 \text{ en dat is juist.}$$

$$\omega = -\frac{1}{2} : 2 \cdot -\frac{1}{2} = 2 + \sqrt{2 \cdot -\frac{1}{2} + 10} \text{ levert: } -1 = 2 + 3 \text{ en dat is onjuist.}$$

Conclusie:  $\omega = 3$  is de enige oplossing van deze vergelijking.